while Onsager symmetry requires $\beta = \tilde{\beta}$.

Let us now consider a single interface from which evaporation into an infinite half-space takes place. In order to determine whether supersaturation upon evaporation is allowed by thermodynamics, we rewrite Eq. (5) using

$$p_e(\bar{T}) = p_e(T_1) + \frac{1}{2}(T_2 - T_1)\left(\frac{dp}{dT}\right)_{\text{sat}} \tag{7}$$

to yield in combination with (4)

$$p - p_e(T_1) = \frac{T}{\rho_2^{-1} - \rho_1^{-1}}(\beta q_{21} + \gamma j_2) + \frac{1}{2}T^2(\alpha q_{21} + \beta j_2)\left(\frac{dp}{dT}\right)_{\text{sat}}. \tag{8}$$

We assume a stationary state, with the heat of vaporization supplied solely through the liquid. Hence one has $q_1 = 0$ and thus $q_{21} = \frac{1}{2}q_2$. Since $q_2 = hj_2$ with $h = L/M$ the enthalpy difference per unit mass, the right-hand side of Eq. (8) can be written in terms of the mass flux j_2 only. If ρ_2^{-1} is neglected with respect to ρ_1^{-1}, and $(T/p)dp/dT = L/RT$ is used, the condition for supersaturation $p - p_e(T_1) > 0$ yields

$$\alpha > 4\gamma/h^2, \tag{9}$$

which can be realized without violating the general conditions imposed by the relations (6). Note that this result is consistent with the condition for "intersecting lines" in the experiment proposed by Waldmann.[5]

It should be emphasized that this derivation by no means proves that supersaturation and the "inverse temperature gradient" will indeed occur. It only shows that this cannot be excluded on the basis of nonequilibrium thermodynamics.

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The eddy viscosity in a turbulent boundary layer on a cylinder

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The eddy viscosity for a turbulent boundary layer on a cylinder was measured using hot-wire anemometry. Except near the wall, the eddy viscosity is roughly constant with a value corresponding to that estimated from the slope of the mean velocity profile. These results confirm that a constant eddy viscosity closure scheme is appropriate for the boundary layer on a cylinder.

Recently, a mixed-scale relation [Eq. (1)] for the logarithmic portion of the mean velocity profile of the turbulent boundary layer on a cylinder in axial flow[1] was proposed:

$$U_+ = (1/m)\ln y_+ + n. \tag{1}$$

The mean velocity $\bar{U}$ and the distance from the wall of the cylinder y are nondimensionalized with wall scales U_τ (friction velocity) and ν (kinematic viscosity) and are denoted with a subscript $+$. Although the form of Eq. (1) is similar to the log law of the wall for a planar, turbulent boundary layer, the coefficients of the log law [Eq. (1)] are not constant. Instead, m and n are functions of δ/a, where δ is the boundary layer thickness and a is the radius of the cylinder.[1]

To derive Eq. (1), it was necessary to assume a constant eddy viscosity closure such that $\tau = \epsilon(d\bar{U}/dr)$, where τ is the total shear stress, ϵ is the eddy viscosity, and r is the radial coordinate. The purpose of this Brief Communication is to show that the eddy viscosity is indeed constant by providing results of the direct measurement of the eddy viscosity in a cylindrical boundary layer. The eddy viscosity is given by $\epsilon = -\rho\, \overline{u'v'}/(d\bar{U}/dr)$. Here ρ is the density; u' is the streamwise velocity fluctuation; v' is the fluctuation in the velocity perpendicular to the wall.

The experimental setup was the same as that described in Ref. 1 (series B). A 0.475 cm cylinder was suspended along the center line of a wind tunnel. The measurements of

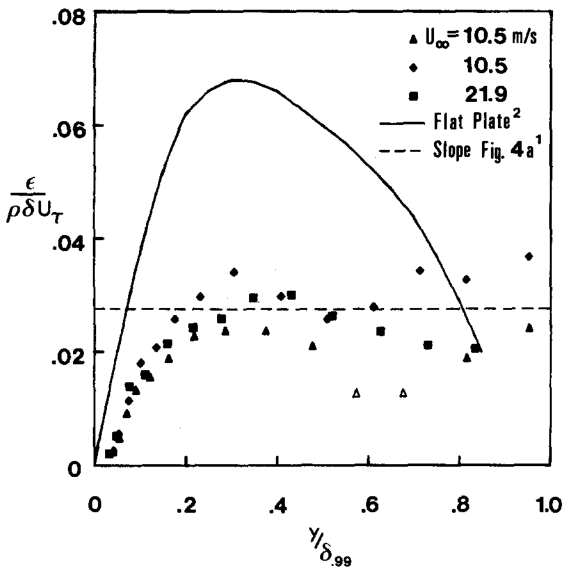

FIG. 1. Profile of the nondimensional eddy viscosity. [$y/\delta = 1.0$ corresponds to $y_+ \approx 590$ for $U_\infty = 10.5$ m/sec ($\delta/a \approx 7.8$) and $y_+ \approx 1160$ for $U_\infty = 21.9$ m/sec ($\delta/a \approx 7.6$)]. The two open data points appear low because of abnormally low measurements of the Reynolds stress.

the Reynolds stress, $-\rho\,\overline{u'v'}$, were made with a home-built X probe with a box size of 0.5 mm (0.02 in.) or about 16 viscous length units at a free-stream velocity of $U_\infty = 10.5$ m/sec. The calibration method was the same as that used in Ref. 1. The mean velocity profile was measured using a U probe with a length of $l = 0.25$ mm (0.010 in.) or about eight viscous length units at $U_\infty = 10.5$ m/sec. Because of the sensitivity of the numerical differentiation scheme to slight errors in the raw velocity measurements, the mean velocity profile was smoothed. Then, evenly spaced estimates of the mean velocity were found via interpolation, and the derivative $d\bar{U}/dr$ was found using central differences.

The nondimensional eddy viscosity is shown in Fig. 1 for two different free-stream velocities. The solid curve on the figure is the eddy viscosity for the planar boundary layer.[2] Compared to the eddy viscosity for a flat plate boundary layer, the eddy viscosity in the cylindrical boundary layer is constant, to a first approximation.

In Ref. 1, it was assumed that Clauser's estimate[3] for a constant eddy viscosity in the wakelike outer region of a planar boundary layer is valid, so that $\epsilon = c\rho\delta U_\tau$, except near the wall. Using this estimate, it was shown that $m = c\delta/a$, where c is a constant. Thus, the slope c of m vs δ/a [Fig. 4(a) in Ref. 1] is simply the nondimensional eddy viscosity. Using linear regression for the data in Fig. 4(a) of Ref. 1 and excluding points for $\delta/a > 40$, results in $\epsilon/\rho\delta U_\tau = 0.0274$. This is shown as the dashed line on Fig. 1. The eddy viscosity estimated from the slope of the mean velocity profile agrees with the directly measured eddy viscosity.

Although the cylindrical boundary layer has a logarithmic mean velocity profile, this log region is unrelated to the planar log region for large δ/a. The cylindrical boundary layer's log region is wakelike in character. In other words, the wall does not constrain the eddy structure, except near the wall. The analysis leading to Eq. (1) is independent of the concepts of a planar log region or a planar outer wake region. The logarithmic velocity profile for a cylindrical boundary layer is a direct consequence of the equations of motion and the constant eddy viscosity closure. The direct measurements of the eddy viscosity presented in this communication indicate that the eddy viscosity is indeed constant, to a first approximation, except near the wall. This confirms the validity of the constant eddy viscosity closure used by Lueptow *et al.*[1]

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